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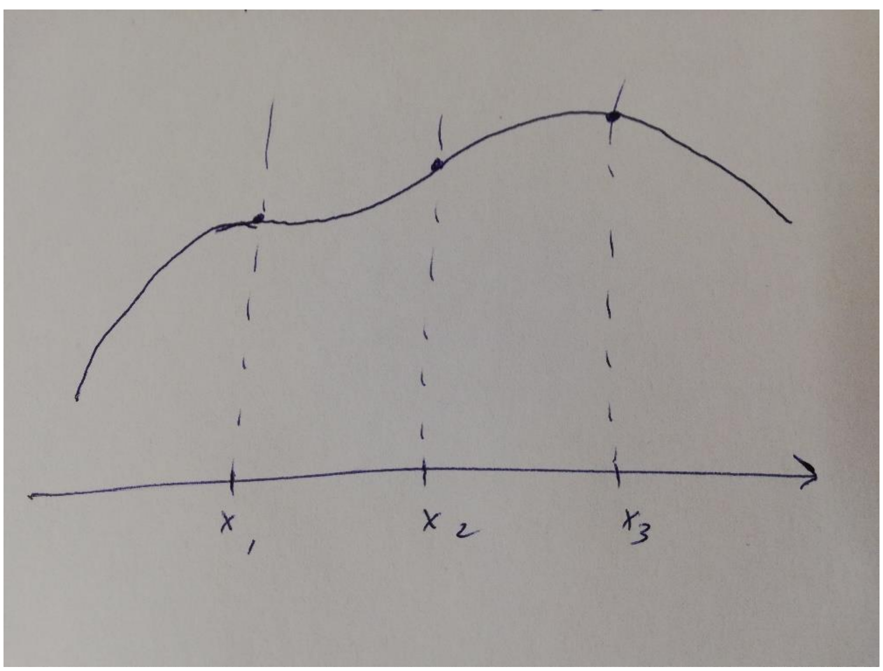
<Course number>

<The date of the paper>

Math Problems

PROBLEM #1

We know that the function is increasing on the intervals, where the first derivative is positive, and decreasing on the intervals, where it is negative. We also know that the function is concave upward, when the second derivative is negative, and concave downward, when the second derivative is positive. Therefore, the graph may have the following form:



a)

The graph of my function has a local maximum at x3. And any graph sketched with this information will have a local maximum at this point, because the first derivative changes its sign from plus to minus when passing through x3.

b)

The fastest increase is on (x1, x2). And any function of this kind will have the fasted increase in here, because this is the only interval when the function is concave downward.

PROBLEM #2

$$f(x) = x^3 - 3x^2, -1 \leq x \leq 3$$

a)

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

b)

Find the critical points, putting the first derivative equal to 0:

$$f'(x) = 3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x_1 = 0$$

$$x_2 = 2$$

There are two critical points, 0 and 2.

c)

The inflection points of f are those, where the second derivative is zero:

$$f''(x) = 6x - 6 = 0$$

$$6(x - 1) = 0$$

$$x = 1$$

There is only one inflection point x = 1.

d)

The evaluation of f at its critical points and at the endpoints is as follows:

$$f(-1) = -1 - 3(1) = -4$$

$$f(0) = 0$$

$$f(2) = 8 - 3 * 4 = -4$$

$$f(3) = 27 - 27 = 0$$

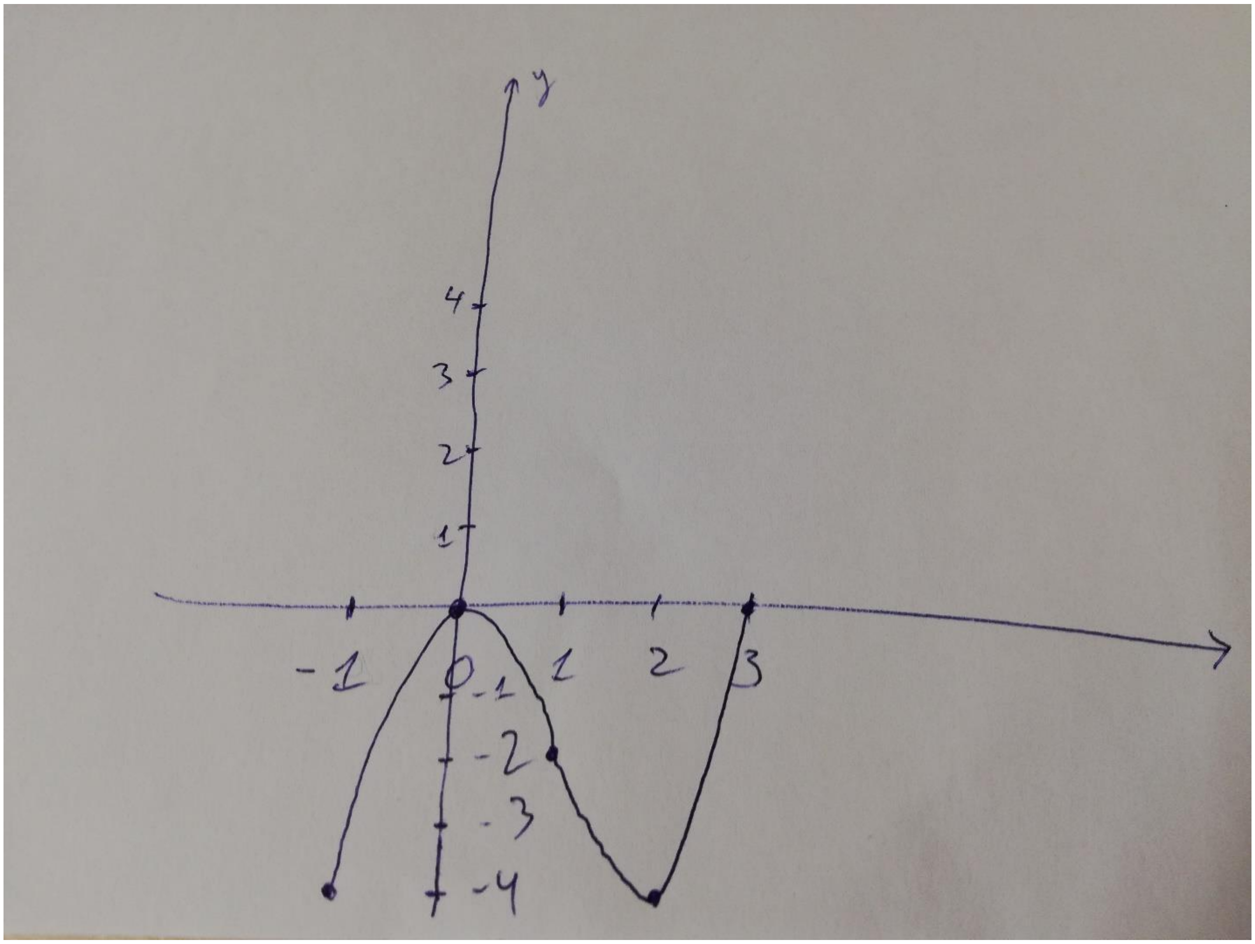
Consider critical point x = 0. The first derivative changes its sign from positive to negative. Therefore, this is the point of a maximum. Consider x = 2. The derivative changes its sign from minus to plus. Therefore, x = 2 is a point of minimum. Since there are two points only, both of them are global.

e)

Find f(1) for our convenience:

$$f(1) = 1 - 3 * 1 = -2$$

Using the information above, we can sketch the graph of the function:


y
4
3
2
1
-1
0
1
2
3
-1
-2
-3
-4